Growth and inequality: access to advanced education under credit constraint

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Abstract

This Master thesis starts by reviewing a wide strand of the literature about the linkage between growth and inequality. It then proposes an overlapping generation model with altruistic agents that are heterogeneous in ability and initial wealth. They go through a two-stage education process, during which the decision of investing in higher education is left to them. Possibility for borrowers to evade debt payments imposes a constraint on credit which will impact human capital accumulation. Quality of the two educational sectors (basic and advanced) depends on public funding which is allocated to each sector. We analyze the impact of initial distribution of wealth on the subsequent achievements of the economy as well as the possible policy recommendations as regards the allocation of resources between the educational sectors.

Contents

1 Introduction

The linkage between growth and inequality has become a major point of interest for economists with a wide field for policy recommendations, all the more since the human capital literature as well as the one on the economics of education has entered the debate. The crucial question is to know whether inequality in the distribution of wealth spurs or slows down economic growth. While it was thought that inequalities, allowing to channel resources towards high saving agents, were impacting positively on growth in a setup of strictly physical accumulation, another thesis has appeared with the integration of human capital accumulation. Indeed, building on the evidence that education of children was somehow positively correlated to the parents' income, many authors have focused on the idea that human capital accumulation was curbed by the fact that low-income households were constrained in their education investment by an imperfect credit market. The result is that, more equality in the distribution of initial wealth allowing to overcome the credit constraint, human capital is accumulated faster and more efficiently in societies that start off with a more equal distribution of wealth and, therefore, these societies achieve higher levels of long-run growth. This result implies important policy recommendations insofar as public funding of education can be a tool in order to ease the credit constraint.

Once this is observed, it is natural to wonder how educational investment exactly is constrained. Indeed, in many countries, primary education is free and often mandatory. It appears then that the constraint weighs on the higher education investment, when tuition fee and opportunity cost of working instead of studying become really significant. Hence the idea of separating basic and advanced education came through, all the more than the way to allocate public funding between the two sectors represents a serious policy concern. A rationale for public funding of education often found in developing countries is to create an efficient advanced education sector able to train highly skilled agents who will fill high-ranking positions in the government, public agencies and private companies from the country. This view is based on the so-called trickle down effect, which argues that developing countries lack efficient governing bodies and that, therefore, investing in the training of an elite will benefit to the rest of the society. An alternative view consists in saying that reallocating resources from basic to advanced education worsens the quality of basic education, which will impact negatively enrollment in higher education and thus will curb human capital accumulation (we will say: the occupational effect).

After a review of the major strands of the literature on inequality and growth, we will construct a three-period life overlapping generation model with agents characterized by altruism, as they derive utility from bequeathing, randomly assigned abilities, which correspond to a capacity to study efficiently, and an initial endowment, received as a bequest. The credit market is imperfect and education is a two-stage process whose quality depends on the allocation for the two sectors. We show that economic achievements are correlated to the initial distribution of wealth and that a more equal distribution is more likely to allow higher levels of growth. Furthermore, we will try to disentangle trickle-down and occupational effects when the government reallocates resources from basic to higher education.

2 Literature review

A very abundant literature has examined the linkage between inequality and growth. Usually building on the empirical evidence that there exists a correlation between inequality in the distribution of wealth within a society and the levels of growth achieved by this economy (it is shown that slowly growing countries exhibit higher inequality levels), this literature does away with the old conjecture according to which inequality would allow higher overall saving rates and thus would spur economic growth. On the contrary, many papers show that, once integrated human capital as a determinant of economic growth, wealth inequalities can beget inefficiencies in educational investments due to the existence of a credit constraint. For instance, starting from the evidence of a positive correlation between an individual's income and his parents', Loury (1981) states that, in a family based economy characterized by intergenerational altruism (modeled via a bequeathing process that will determine the investment in training of the children) where there does not exist possibilities of interfamily loans or income risk-sharing insurance, there is room for public intervention to outperform the laissez-faire outcome. Loury (1981) assumes that the distribution of abilities in the society is random allowing for highly able children to be born in low-income families where their access to training will be constrained due to a credit market failure. In such a setup, Loury (1981) shows that redistributive taxation is desirable insofar as it allows to achieve a long-run earnings distribution with both higher mean and lower variance than in the laissez-faire outcome. In Loury (1981)'s setup, desirability of an outcome is based on a trade-off between inequality and social mobility captured by the indirect utility function. This way, no use is made of a social welfare function but desirability is rather measured on an individualistic basis. The intuition of the result is then that an egalitarian redistributive taxation system allows somehow an income risk-sharing process. Indeed parents do not know the ability of their offsprings when they decide of the education investment. Redistributive taxation allows to reduce the risk of this investment, which is welfare enhancing insofar as parents are risk-averse. The result according to Loury (1981) is that we obtain a Rawlsian definition of a desirable outcome as it consists in placing oneself behind a kind of veil of ignorance (nobody knows what one's offspring position in the society will be) to decide of the best policy to implement. Loury (1981) however underlines that the randomly attributed abilities represent quite a strong assumption. Indeed, although there is no consensus on the ability transmission process, it cannot be denied that there exists some persistence of abilities within a dynasty due to education and gene transmission, which the model totally does not account for.

Building on the same ground but modeling differently the credit market imperfection (infinite interest rate in Loury (1981) versus monitoring costs here) as well as the ordering of the outcomes, a major paper in this literature was written by Galor & Zeira (1993). The conclusions are globally the same but the setup is quite different. Individuals are still guided by altruism but the credit market, instead of being merely inexistent, exhibits an imperfection in the sense that individual borrowers can default and escape reimbursement. This leads lenders to charge higher interest rates than in a perfect market equilibrium in order to cover the monitoring and default costs. Added to this, a crucial assumption is the non-convexity of the educational investment decision. Indeed, individuals only choose whether to invest or not in human capital (indivisibility of the investment) and it decides of their type: skilled or unskilled. These two types then access two different technologies (only unskilled labour-based or skilled labour and physical capital combined) that will pay different salaries, obviously higher for the skilled. Under such a setup, Galor & Zeira (1993) show that there exists a bequest threshold under which it is preferable for the agent not to invest in human capital and work as an unskilled because borrowing to finance the educational investment would be too costly. Under this threshold, dynasties will converge to an unskilled low wage long-run situation, whereas above it, dynasties will reach a skilled high wage equilibrium. Economies with same preferences and technologies can thus converge towards different equilibria as soon as initial wealth is not distributed the same way. What consequently decides of long-run growth and distribution of wealth will be initial endowments. For a given level of aggregate initial wealth, a society with a larger middle class may achieve a higher long-run growth and a more equal distribution than one that starts with only a few well-off. This idea generalizes to the situation where skills are randomly attributed to every individual. There will be an ability threshold added to the bequest threshold that will prevent too poorly able individuals to invest in human capital and will allow highly able individuals to have a lower bequest threshold. Still, the same conclusion prevails concerning social mobility, except that there will exist an area where upward and downward mobility will take place, without changing the general results. In all cases, we obtain the same results as in Loury (1981) that initial distribution of wealth has an effect on long-run growth and distribution of earnings and that initial inequality impacts negatively as the credit constraint prevents some potentially able individuals to borrow and invest in human capital.

Moav (2002) further extends this model by replacing the non-convexity of the educational investment by a convex saving function. This extension builds on the empirical evidence that saving behaviours (here the bequeathing process) generally exhibit a convex shape. Moav shows that such a bequest function combined with the same credit market imperfection as in Galor & Zeira (1993) is enough to ensure the result that dynasties that are originally poor converge to a poor long-run equilibrium while originally rich dynasties stay rich forever. In this setup again, inequality has a negative impact on long-run growth, at least as long as the initial proportion of families that are below the saving threshold is not too high (so relatively rich families). Indeed, in poor economies, it may happen that inequality has some positive impact on aggregate savings and consequently on output, although not on income. We find back the conjecture quoted earlier according to which inequality spurs economic development via higher overall saving rates. The Kuznets conjecture reconciles these two points of view: it states that the linkage between inequality and growth depends on the stage the economy is in. Formally, less developed countries exhibit high inequality levels but this features fades away as the economy becomes more developed.

Galor & Moav (2004) have built a model in order to illustrate this conjecture. In their model, the prime engine of growth in less developed economies is physical capital accumulation. At this stage, inequality impacts positively economic development by channeling wealth towards individuals with the highest saving rates. Nevertheless, return to human capital capital tends to increase due to skill-capital complementarity and therefore human capital accumulation becomes the prime engine of economic growth in so called developed economies. Once it is the case and in presence of a credit market imperfection, it has already been shown that equality spurs economic growth because it allows more individuals to invest in education and become skilled.

Another strand of the literature has focused rather on the correlation between parent and child occupation and income. This literature builds on the empirical evidence according to which the credit constraint in the US is not binding and that other factors would explain inequalities in the education investment of children coming from different backgrounds. Authors like Heckman insist on the fact that skill formation is a life-long process and particularly on the demonstrated importance of the parents' human capital level as well as early investment in human capital of the children rather than on the credit market imperfection affecting the higher education investment decision. Indeed, Heckman (e.g. Carneiro, Cunha & Heckman 2003) mentions two fundamental features of the skill formation process: first, the malleability at early stages, meaning that the parents' human capital level as well as early investment are crucial for the child's development; second, the complementarity of the human capital investments, meaning that early investment that is not followed by investment in higher education or professional training is not productive and that any lack at the early stage will be extremely costly to catch up later. The idea of randomly assigned abilities is kept but is given much less importance as many other parameters play a role. The result is that the credit constraint is not any more the critical determinant of persisting inequalities and low growth levels. Indeed, as demonstrated in studies of college attendance in the US, the credit constraint is not binding for a great majority of students. Heckman rather blames what he calls a "failure on the parents' market", namely the fact that children cannot buy their parents and thus fully decide of their human capital investment.

An interesting attempt to reconcile these strands of the literature but in a quite peculiar setup is due to Lloyd-Ellis (2000). Indeed, Lloyd-Ellis is as well interested in drawing recommendations for policy makers regarding the growth-inequality relationship but, rather than studying the impact of a credit constraint, he designs a dual labour sector and a peculiar schooling system that will exhibit a non-convexity. In fact, every individual is randomly assigned an ability level, receives a bequest from his parents and basic education from the State, whose quality depends on the level of public spending in this sector. From there, individuals chooses whether to go on with further studies and therefore invest in higher education (whose quality depends on State funding) or rather directly start working. The non-convexity appears because the investment in higher education is discrete (choice of investing or not). Two kinds of workers offer their services: employees, whose wage depends on the overall productivity (for which the human capital of all workers counts) and managers, who get paid according to their own individual characteristics. There consequently exists a wedge between private and social return to schooling for certain workers. This model exhibits a negative correlation between inequality and growth. Indeed, a mean-preserving reduction in income dispersion will raise the average efficiency of the parental investment in the children schooling (due to decreasing marginal returns of parental inputs), this is the so-called productivity effect. It will as well raise the fraction of low wage children entering higher education, impacting positively the supply of skilled workers and consequently aggregate output (the occupational effect). This setup allows to evaluate the effect of reallocations of State funding across basic and higher education. Reallocating towards higher education, which is often done in developing countries based on the argument that it will allow to fill governing positions efficiently and thus enhance overall productivity (the trickle down effect), will in fact reduce quality of basic education and consequently exclude agents with relatively low human capital potential from entering higher education. It will increase inequality which is a burden on growth in this model. Here, we find again the idea of Heckman according to which a low early stage investment is too costly to be caught up later on and that, at the same time, early investment not followed by higher education is useless.

3 The model

3.1 The setup

3.1.1 The households

We consider an overlapping generation model. Agents in this economy are heterogeneous as they differ in their initial wealth and in their level of ability. Abilities in this model amount to a capacity for an agent to benefit efficiently from education. The level of ability is randomly assigned to each agent for her entire life and is noted $a \in [\underline{a}; \bar{a}]$, with $g(a)$ being the density function[1]. The second heterogeneity amongst agents stems from the initial wealth they receive in their first period of life as a bequest from their parent. Bequest received will be noted $b_t \in [\underline{b}; \bar{b}]$, with $f_t(b_t)$ being the density and the subscript t being the index for periods.

Agents live for three periods. The first one is dedicated to mandatory basic education while the second one offers a choice between going on with further studies (we will use the term: higher education) or working right away. In the third one, everybody works, consumes and bequeaths. Each agent in its third period, that we will call parent, is assigned a newly born agent, that we will call child, and to whom she will leave a bequest. Agents are thus considered to be altruistic as they derive utility from bequeathing. The utility function takes the following form:

$$U_t = (1-\rho)\log C_t + \rho \log b_{t+1} \tag{1}$$

Where C_t is the consumption of agents in their third stage in period t and b_{t+1} is the parent's bequest to her child also in period t. ρ represents the altruism coefficient.

The succession of parents and children will be called a dynasty. Notice that, this way, there is no population growth.

The accumulation of human capital via education is divided into two stages and is therefore a sequential process. It might be the case that the ability level of an agent is so low that human capital accumulated after the first period is not enough to make investment in higher education worthwhile. Such an agent thus prefers to work right away and we will call her "unskilled worker". It might as well be the case that the agent initial wealth is not enough to cover the higher education private cost (noted ϕ). By private cost, we mean mainly the tuition fee plus as well the cost of living of the second

[1]It is true that randomly assigned abilities might be considered to be a strong assumption as one cannot deny that there exists some kind of ability transmission process from parents to offsprings both via genes and education. Nevertheless, the features of this process are still largely disputed and not strongly ascertained, that is why we stick to the random assignment here. We will comment further on that in the extension section

period, cost that is not an obstacle for unskilled workers (because they work in second period) but which must be covered by initial wealth or borrowed for agents that wish to enter higher education. If neither ability nor initial wealth is a problem, agents invest in higher education, accumulate further human capital and work in their last period of life as "skilled" workers.

How much human capital they accumulate will depend on the agent's ability, on whether she enters higher education and on public spending in the basic and higher education sectors. We will note h_{Bt} and h_{At} the level of human capital of an agent with ability a respectively after basic and advanced education. The formal relationship is the following:

$$\begin{cases} h_{Bt} = aE_{Bt}^{\alpha} & \alpha \in [0;1] \\ h_{At} = h_{Bt}E_{At}^{\beta} & \beta \in [0;1] \end{cases} \tag{2}$$

where E_{Bt} and E_{At} represent the quality of respectively basic and advanced education proxied by the quantity of public funding invested in each sector.

3.1.2 The government

As seen previously while exposing the human capital accumulation process, the government plays a crucial role in that it decides of the quality of the two educational sectors by allocating resources among them. Another crucial step is to decide of the taxation rate τ that will give the total amount of public funding to spend on education. For this decision, the government has the following budget constraint:

$$E_{Bt} + E_{At} = \tau Y_{t-1} \tag{3}$$

Where Y_{t-1} is the aggregate production in period t_1 (and therefore the aggregate income as well).

Then, considering that e_B is the share that the government dedicates to basic education (we will leave this share constant), we have the following relationships:

$$E_{Bt} = e_B \tau Y_{t-1} \tag{4}$$

$$E_{At} = (1 - e_B)\tau Y_{t-1} \quad (5)$$

3.1.3 The credit market

We consider that the credit market is imperfect for agents[2] in the sense of Galor & Zeira (1993). Agents in this setup have the possibility of evading repayments by hiding or fleeing but it is costly to them. In order to avoid defaults, lenders can keep track of their borrowers but at a cost that will impact the interest rate for agents. We assume that it is less costly for lenders to keep track of the borrowers than it is for borrowers to evade debt payment. It results that a credit market will exist but that the interest rate for agents, noted i, will be higher than the worldwide interest rate, noted r (we take the case of a small open economy, where the worldwide interest rate prevails).

Together with the non-convexity in the educational investment (indivisible investment), the credit market imperfection is crucial for the results here to hold. The idea is that it may happen that highly able agents born to a poor family choose not to enter advanced education, which represents an inefficiency.

3.1.4 The production side

We assume that the economy only produces one good thanks to a Cobb-Douglas technology combining physical and human capital. The production function can be stated as follows:

$$Y_t = AK_t^{\gamma} H_t^{1-\gamma} \quad \gamma \in [0;1] \quad (6)$$

Where A is the total factor productivity, K_t and H_t are respectively the stocks of physical and human capital in period t.

We define H_t as the integral of the individual human capital stock over the whole working population, both skilled and unskilled. As we are in a small open economy, firms can rent physical capital at the world interest rate r.

[2] Firms do not suffer from the credit constraint

Hence, assuming perfect competition, both factors will be paid their marginal productivity. We therefore have the following regarding the relationship between worldwide interest rate and the derivative of the production function with respect to K_t:

$$F_{K_t} \quad = \quad r \quad = \quad \gamma A \frac{K_t}{H_t}^{\gamma-1} \tag{7}$$

Which can be rewritten as:

$$\frac{K_t}{H_t} \quad = \quad \left(\frac{r}{\gamma A}\right)^{\frac{1}{1-\gamma}} \tag{8}$$

Which is a constant.
As the human capital stock is known one period in advance (because it results of investment decisions that take place at the end of the previous period), firms can adjust perfectly their demand of physical capital in order to keep the physical to human capital ratio unchanged. Taking the derivative of the production function with respect to H_t now gives us an expression for the remuneration of a unit of human capital:

$$\begin{aligned} F_{H_t} \quad = \quad w \quad = & \quad (1-\gamma)A(\tfrac{K_t}{H_t})^{\gamma} \\ = & \quad (1-\gamma)A(\tfrac{r}{\gamma A})^{\frac{\gamma}{\gamma-1}} \quad \text{Using (8)} \end{aligned} \tag{9}$$

We can see from there that remuneration depends on how much human capital one can offer. We can therefore deduce that unskilled and skilled workers will have two different wage functions insofar as they do not have the same amount of human capital to offer. But not only will the wage function depend on the type of education an agent chose but as well it will be directly correlated to the ability level of the agent. Let us derive the explicit wage functions:

$$w^u \quad = \quad waE_{Bt}^{\alpha} \tag{10}$$

$$w^s \quad = \quad waE_{Bt}^{\alpha}E_{At}^{\beta} \tag{11}$$

Where w^u and w^s are respectively the wage functions for unskilled and skilled workers.

As we can see from these expressions, there exist two types of wage function and consequently a continuum of wages with a jump while reaching an ability level enough to enter advanced education.

3.2 Optimal behaviour

Let us consider an individual with ability $a \in [\underline{a}; \bar{a}]$ and bequest received from her parent b_t. This individual decides of her consumption C_t and of her bequest to her offspring b_{t+1} under the constraint that it does not overwhelm total wealth, that we will note W_t. Total wealth depends on the second stage decision: either one has not entered advanced education and thus has lent her whole bequest, worked right away as an unskilled and for the two last periods, either one has entered advanced education and needed to borrow to do so, she will then work as a skilled in the last period while repaying her loan, either finally one has entered advanced education and could pay the private cost from the bequest, in which case she has lent the rest of the money and works as a skilled in the last period while being repaid her loan. The individual's maximization problem can be stated as follows:

$$\begin{aligned} \max_{C_t, b_{t+1}} U_t &= (1-\rho)\log C_t + \rho \log b_{t+1} \\ \text{s.t.} \qquad W_t &= C_t + b_{t+1} \end{aligned} \tag{12}$$

Deriving the first order conditions, it is not hard to obtain the following:

$$C_t = (1-\rho)W_t \tag{13}$$

$$b_{t+1} = \rho W_t \tag{14}$$

As already explained earlier, total wealth W_t depends on second stage decisions and we can distinguish between three situations:

1. The agent has not entered advanced education, has lent the full bequest and has worked during the two last periods as an unskilled. Here

is the expression W_t^u for total wealth in this case:

$$W_t^u = (1+r)^2 b_t + (1-\tau)(2+r)w^u \tag{15}$$

2. The agent has entered advanced education but has borrowed in order to pay the private cost ϕ, she then pays back her loan as she works as a skilled in the last period. This is an expression for W_t^b (total wealth of borrowers):

$$W_t^b = (1+i)((1+r)b_t - \phi) + (1-\tau)w^s \tag{16}$$

3. The agent has entered advanced education and had enough money from the bequest to pay the full private cost ϕ right away. She could thus lend what was left and get debt payments in addition to her skilled wage during the last period. Here comes the expression for W_t^r (total wealth of rich individuals):

$$W_t^r = (1+r)((1+r)b_t - \phi) + (1-\tau)w^s \tag{17}$$

From this set of expressions, we can deduce conditions on the parameters that will govern the second stage decisions of the agents. Indeed, we can draw from equations (15) and (16) that borrowers will decide to invest in higher education if the utility they derive from it is higher than from the unskilled situation, and this is the case whenever $W_t^b > W_t^u$. Let us rewrite this condition to obtain an initial wealth threshold $b_t^*(a)$ under which individuals choose not to invest:

$$b_t^*(a) = \frac{(1+i)\phi - (1-\tau)waE_{Bt}^{\alpha}[E_{At}^{\beta} - (2+r)]}{(1+r)(1+i)} \tag{18}$$

It is straightforward from the formula to see that this threshold increases with the advanced education private cost ϕ. We can even add that it goes down as the agent's ability rises and as total public expenditures increases provided we make the following assumption, for any given agent with ability

a:

$$(2+r)w_t^u \quad < \quad w_t^s \tag{19}$$

Which means that any agent would earn more working one period as a skilled rather than two period as an unskilled.
From equations (15) and (17), we can compute an ability level a^* under which lenders choose not to invest in advanced education. They choose so because utility being an unskilled is higher than the one derived from becoming skilled. This happens whenever $W_t^u > W_t^r$. Rewriting this inequality, the ability threshold takes the following expression:

$$a^* \quad = \quad \frac{(1+r)\phi}{(1-\tau)wE_{Bt}^{\alpha}[E_{At}^{\beta}-(2+r)]} \tag{20}$$

From the thresholds expressed in (18) and (20), we can deduce the fraction of population, say S_t, that will become skilled. We will have:

$$S_t \quad = \quad \int_{a^*}^{\bar{a}} \int_{b_t^*(a)}^{\bar{b}} f_t(b_t)\ g(a)\ db_t da \tag{21}$$

We can deduce from this expression that the size of the skilled population depends on the distribution of individual abilities and on the distribution of initial wealth.

3.3 Intergenerational mobility

3.3.1 The rise and fall of dynasties

[3]

In the previous section, we have derived the short-run optimization results. In this section, we focus on the dynamics of intergenerational mobility and on its long-run implications. As seen in the previous section, the distribution of total wealth in period t depends on the distribution of abilities as well as on the distribution of initial wealth (that is of the bequests left

[3] as labeled by Becker & Tomes (1986)

by the previous generation). We can therefore deduce a dynamic system of equations giving b_{t+1} in function of b_t and a.

$$b_{t+1} = \begin{cases} \rho((1-\tau)(2+r)w^u(a) + b_t(1+r)^2) & \text{if} \quad b_t < b_t^* \quad \text{or} \quad a < a^* \\ \rho((1-\tau)w^s(a) + (1+i)(b_t(1+r)-\phi)) & \text{if} \quad b_t^* \leq b_t < \frac{\phi}{1+r} \quad \text{and} \quad a \geq a^* \\ \rho((1-\tau)w^s(a) + (1+r)(b_t(1+r)-\phi)) & \text{if} \quad b_t \geq \frac{\phi}{1+r} \quad \text{and} \quad a \geq a^* \end{cases} \tag{22}$$

Where b_0 is exogenously given.

Thus defined in (22), we have a Markov process giving b_{t+1} in function of b_t conditionally on the ability. We can see that, below the threshold a^*, b_{t+1} in function of b_t is an affine transformation with a small coefficient $\rho(1+r)^2$ whereas, above this threshold, b_{t+1} in function of b_t exhibits kinks. For those born poor (that is with $b_t < \frac{\phi}{1+r}$, meaning that they need to borrow if they want to enter advanced education), we have to distinguish between those who will remain unskilled and those who will borrow to get advanced education. For the latter, the coefficient will be $\rho(1+r)^2$ as well while, for the former, it will be $\rho(1+i)(1+r)$ which is bigger (as $i > r$ by assumption). Here we observe a kind of leverage process for those who can borrow: it is costly but the returns are worth it. Finally, for those born rich (that is with $b_t \geq \frac{\phi}{1+r}$, meaning that they can afford the private cost of advanced education without borrowing), the coefficient is back at $\rho(1+r)^2$. Hence, if we want to picture this conditional Markov process, we have a continuum of parallel straight lines with coefficient $\rho(1+r)^2$ for individuals whose ability ranges from $\underline{a}$ to a^* and, starting from a^*, we have parallel lines but exhibiting two kinks: one in b_t^* and another one in $\frac{\phi}{1+r}$. The coefficient outside these boundaries is $\rho(1+r)^2$ as well and it is $\rho(1+i)(1+r)$ on the inside.

In order to derive some stability results, we need two further assumptions, which follow:

$$\rho(1+r)^2 \quad < \quad 1 \tag{23}$$

$$\rho(1+i)(1+r) \quad > \quad 1 \tag{24}$$

The first restriction prevents bequests from growing indefinitely whereas the second one implies that the credit constraint is significant, that is that the cost of keeping track of the borrowers is high enough to have a significant spread between the world interest rate and the one applied to agents.
Once stated these restrictions, we can focus on the some stable points that the Markovian system exhibits. By this, we mean the points that are left unchanged by the bequeathing process, that is that individuals bequeath exactly the same amount they were bequeathed. We can calculate these points setting: $b_{t+1} = b_t$, which will give us three functions of a.

- Let us take the case of agents with abilities ranging from $\underline{a}$ to a^*. We have a unique function of a $b_B(a)_{a<a^*}$ that verifies $b_{t+1} = b_t$, it takes the following form:

$$b_B(a) \quad = \quad \frac{\rho(1-\tau)(2+r)waE^{\alpha}_{Bt}}{1-\rho(1+r)^2} \tag{25}$$

  As $(1+r)^2 < 1$, we can infer that individuals that were endowed with a bequest smaller than $b_B(a)$ will bequeath more and, on the contrary, those who had more will bequeath less. We can even say that, for dynasties of families with abilities below a^*, the bequeathing process will converge towards the continuum of long-run values given by $b_B(a)$.

- We now turn to the case when abilities range from a^* to $\bar{a}$. Given the properties that we exposed earlier about the relationship between b_{t+1} and b_t in this case and given the restrictions (23) and (24), we will have three functions of a that will leave the bequests unchanged across generations: one that is smaller than b^*_t, one between b^*_t and $\frac{\phi}{1+r}$, and one bigger than $\frac{\phi}{1+r}$.

  The first one takes the same form as $b_B(a)$ in (25).

The second one is as follows:

$$k^*(a) \quad = \quad \frac{\rho[(1-\tau)w\, a\, E_{Bt}^{\alpha}\, E_{At}^{\beta} - (1+i)\phi]}{1-\rho(1+r)(1+i)} \tag{26}$$

Finally, the last one is given by:

$$b_A(a) \quad = \quad \frac{\rho[(1-\tau)w\, a\, E_{Bt}^{\alpha}\, E_{At}^{\beta} - (1+r)\phi]}{1-\rho(1+r)^2} \tag{27}$$

It is again quite simple to infer the dynamics of such a process: having received a bequest below $b_B(a)$, an individual will bequeath more; for a bequest between $b_B(a)$ and $k^*(a)$, an individual would bequeath less; inversely if the bequest received is between $k^*(a)$ and $b_A(a)$; finally, an individual with initial bequest above $b_A(a)$ will bequeath more. Therefore we can say that dynasties starting off with less than $k^*(a)$ will converge towards the continuum of long-run values given by $b_B(a)$. Those starting with more than $k^*(a)$ will converge towards the set of long-run values given by $b_A(a)$ provided abilities stay above a^*. Here we would like to emphasize that there will exist individuals that enter advanced education but will bequeath less than what they received so that one offspring at some point will drop advanced education and thus will end up in a poor dynasty, namely they are those who received a bequest between b_t^* and $k^*(a)$. On the other hand, individuals born poor (less than $\frac{\phi}{1+r}$) but who received though more than $k^*(a)$ will enter as well advanced education and bequeath more than what they received allowing their offsprings to become rich ($b_A(a) > \frac{\phi}{1+r}$) provided they keep abilities higher than a^*.

To sum up this process, we can say that the society is divided in two classes: skilled and unskilled. Whether one belongs to one or the other depends on the distributions of abilities and initial wealth. While those born with less than $k^*(a)$ are sure that the dynasty of their offsprings will finally converge towards poor long-run values and an unskilled situation, those born with more than $k^*(a)$ have pretty good chances to see their dynasties converge towards the high long-run values and a skilled situation provided all the successive offsprings maintain at least an ability equal to a^*.

3.3.2 Interclass mobility across generations

We mean by interclass mobility across generations the fact that dynasties may change class over generations depending on their successive levels of ability and initial wealth. We observe two kinds of mobility: upward and downward. The latter concerns dynasties that start off poor (less than $\frac{\phi}{1+r}$) and though manage to become rich through investment in advanced education. The only way to succeed is to have ability levels that stay over a^* and at the same time an initial wealth above $k^*(a)$. As regards the former, it concerns on the contrary dynasties that start off rich and fall into poverty. For that to happen, it must be the case that at some point in the dynasty an individual is endowed with an ability below a^* so that she does not invest into advanced education, bequeath to her offspring less than what she received and possibly less that $k^*(a)$.
We can now compute a transition conditional matrix of this process:

		Child	
		rich	*poor*
Parent	*rich*	Prob(r/r)	Prob(p/r)
	poor	Prob(r/p)	Prob(p/p)

Where:

- Prob(r/r) gives the probability that agents born to rich parents remain rich. Being born rich, they necessarily have more than $\frac{\phi}{1+r}$ $(> k^*(a^*))$, they therefore only need to have an ability level above a^* to remain rich. Prob(r/r) takes the following expression:

$$Prob(r/r) = \int_{a^*}^{\bar{a}} \int_{\frac{\phi}{1+r}}^{\bar{b}} f_t(b_t)\, g(a)\, db_t da \tag{28}$$

- Prob(p/r) gives the probability that agents born to rich parents become poor. For this, it suffices that a rich agent gets an ability endowment lower than a^*, provided that we make the following assumption:

$$\rho((1-\tau)(2+r)w^u(a^*) + \bar{b}(1+r)^2) < \frac{\phi}{1+r} \tag{29}$$

  This assumption makes sure that even the richest individual, if of type a^* or less, will end up poor. We will call this process downward mobility. We have the following form:

$$Prob(p/r) = \int_{\underline{a}}^{a^*} \int_{\frac{\phi}{1+r}}^{\bar{b}} f_t(b_t)\, g(a)\, db_t da \tag{30}$$

- Prob(r/p) gives the probability that agents born to poor parents become rich. To compute this probability, we need to find the bequest threshold $j^*(a^*)$ lying in between $k^*(a^*)$ and $\frac{\phi}{1+r}$ above which individuals with ability of a^* and more will bequeath at least $\frac{\phi}{1+r}$. Here is what $j^*(a^*)$ verifies:

$$\rho((1-\tau)w^s(a) + (1+i)(j^*(a^*)(1+r) - \phi)) > \frac{\phi}{1+r} \tag{31}$$

  Now we can express formally Prob(r/p), which we will call upward mobility, as follows:

$$Prob(r/p) = \int_{a^*}^{\bar{a}} \int_{j^*(a^*)}^{\frac{\phi}{1+r}} f_t(b_t)\, g(a)\, db_t da \tag{32}$$

- Prob(p/p) gives the probability that agents born to poor parents remain poor. For this to happen, there are two possibilities: either the individual has an ability below a^* and she will therefore remain poor for sure, either she has an ability above a^* and she will remain poor if she has an initial wealth below $j^*(a^*)$. Here is the form that Prob(p/p) takes:

$$Prob(p/p) \quad = \quad \int_{\underline{a}}^{a^*} \int_{\underline{b}}^{\frac{\phi}{1+r}} f_t(b_t)\, g(a)\, db_t da + \int_{a^*}^{\bar{a}} \int_{\underline{b}}^{j^*(a^*)} f_t(b_t)\, g(a)\, db_t da \tag{33}$$

3.4 Main results

3.4.1 Impact of the initial distribution of wealth

It is very useful to see the probabilities we just computed as the expected fractions of population that will be mobile or not. An important result to draw from these probabilities is that depending on the initial distribution of wealth as well as on the distribution of abilities, it is more than likely that we will have multiple possible long-run distributions. Indeed, the distribution of a around a^* will be crucial to decide of the extent of the overall investment in human capital in this economy. Moreover the initial distribution of wealth will have a huge impact on the way the economy is going to behave insofar as it will determine how many individuals are locked from the beginning in the unskilled/poverty trap, that is those who got less than $k^*(a^*)$ in the very first period. It is then quite straightforward to observe that, if we consider two economies (same preferences, same technologies, same overall wealth) that differ only in the distribution of initial wealth, the one with the smallest variance is more likely to achieve higher levels of human capital accumulation and thus wealth (although the random abilities might blur the picture), because less agents are likely to get trapped into unskilled positions. The important result here lies in that we have a persistent impact of initial distribution of wealth on the subsequent economic achievements and that more equality in the initial distribution, helping to overcome the credit constraint, means higher growth levels.

3.4.2 Reallocation of resources between educational sectors

In order to address the question presented in introduction on the optimal allocation of resources among the two sectors, we need to disentangle the two effects at work previously described: trickle-down and occupational. Let us come back to what we said in introduction: reallocating resources towards the advanced education sector increases the returns to advanced education and thus the incentive to invest in it, on the other hand, it deteriorates the quality of basic education and therefore worsens the situations of those who cannot invest, which will impact negatively enrollment in higher education of the subsequent generation. In turn, the inverse movement, say reallocation of resources from advanced to basic education, will improve the situation of the unskilled and then relax the liquidity constraint of their offspring. It will nevertheless also decrease the incentive to invest in advanced education. Now that we presented the two ambiguous effects, let us try to see whether one dominates. To do so with our model, we need to take the derivative of the ability and wealth thresholds a^* and $k^*(a)$ with respect to e_B, which is the share of public resources dedicated to basic education. In order for an analytical solution to appear quite simply, we restrict to the case when $\alpha = \beta = 1$. One can check that there exists a e_B^* such that, above (respectively below) e_B^*, a^* and $k^*(a)$ decrease (respectively increase) with e_B^* and we have:

$$e_B^* = \frac{1}{2} - \frac{2+r}{2\tau Y_{t-1}} \tag{34}$$

It means that, unless the share of basic education exceeds e_B^*, the occupational effect dominates so that reallocating from advanced to basic education lowers the thresholds to access higher education. It will result in a higher fraction of individuals investing in higher education and thus a higher growth level. Obviously, the conclusions are opposite if we are above e_B^*.

4 Extensions

In order to speak about possible extensions of this model, one must start with speaking of its weaknesses. As mentioned already, one of the strongest assumption for a model in economics of education is the one of the randomly assigned abilities. Indeed, since very little has been definitively ascertained as regards the transmission of talents and is still subject of strong disputes, it seems a natural assumption to consider that the distribution of these talents is random. However, in order to be precise here, we speak about talent or ability as a capacity to study efficiently, which is different from the managerial talent for instance that is often considered as randomly distributed. Concerning this capacity to study efficiently, no one can deny that there exists a positive relationship between the level of education and income of the parents and those of the children. Be it through genetic transmission, education or early stage educational investment as in Carneiro, Cunha & Heckman (2004), there exists some ability transmission between parents and children. It appears then that we should correct the model in order to integrate these factors in the determination of the ability rather than simply assuming it is random. For instance, we could take the parental level of human capital as a proxy for the ability of the children. Thus it would imply that the ability constraint as well as the liquidity one are determined by one's parents instead of being just random. It would allow to become consistent with the critics that demonstrate that the credit constraint is not binding as regards college attendance in the US and with the thesis of Heckman according to which the credit constraint is not necessarily the crucial problem, but that it is rather the imperfection on the market for parents[4] that creates the inefficiency in the human capital accumulation. It would be all the more satisfying that it would help to solve one of the main caveats of our current model that is the lack of long-run stability. Indeed, the fact that abilities are random makes the mobility process very unstable and it seems very dubious that it is possible to derive a long-run equilibrium at

[4] the fact that children cannot buy their parents on a market and cannot even choose them

this state. In the case where abilities would depend on the parent's level of human capital, we would get much more stability in the behavior of the dynasties and it might therefore be possible to define some long-run equilibria. Moreover, it would very probably be more realistic in the sense that upward and particularly downward mobility would be much more limited. Here, a simple accident in the ability of a high-skilled dynasty member can lock the dynasty in the poverty trap forever. Such a situation would be less likely to happen in the extended model. We thus believe that it would fit better observed facts.

5 Conclusion

After having reviewed different strands of the literature about the economics of education, we have constructed a model based on important features of the models we described in the review. Like almost all the models in this literature, we built an overlapping generation framework. Agents are considered to be altruistic and heterogeneous in initial wealth like in Galor & Zeira (1993). They are assigned randomly abilities, which must be understood in the sense of a capacity to study efficiently, as in Loury (1981). Education is divided into two stages: a basic mandatory one and an advanced one for which investment is needed, as in Lloyd-Ellis (2000). The decision to invest or not is subject to a credit constraint that is constructed as in Galor & Zeira (1993). This framework allowed us to derive that the initial distribution of wealth does matter and determines the human capital accumulation process as well as future economic growth. We could as well infer that reallocating resources from basic to advanced education had two conflicting effects: on the one hand, it increases returns to higher education and reinforces thus the incentive to invest, but on the other hand, it deteriorates the quality of basic education and thus the wage of those who cannot invest in higher education, which, as a consequence, further worsens the liquidity constraint of the next generation. These results were already established in Loury (1981), Galor & Zeira (1993) and Lloyd-Ellis (2000), however not in the same general setup with human capital accumulation and randomly assigned abilities.

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