

Institutions Quality and Growth

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June 2009

Abstract

We analyze the dynamic interaction between civil society organizations and Government in a representative developing economy. Government favors corruption and so fails to build efficient institutions. On its side, civil society exerts pressure on Government to constrain it to halt corruption. We distinguish between an authoritarian Government and an unrestrictive one: the latter does not repress society's protests while the former implements punishment mechanisms. We demonstrate analytically the existence of a unique stable equilibrium by solving a linear quadratic differential game for three Regimes respectively the optimal control problem, noncooperative and cooperative games. Numerical assessment indicates that civil monitoring always increases as corruption increases, but civil monitoring is low and institutions improve much faster under cooperation. Furthermore, total factor Productivity effects always dominate the detrimental effect of civil monitoring on growth in the first regime, under some restrictions in the second and never in the third. In response to a change in the government's aversion to rent variations in the presence of authoritarian government, total factor productivity effects always dominate under both the noncooperative and cooperative scenario.

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1 Introduction

Recently, numerous studies in institutional economics have highlighted the importance of institutions for growth and economic development by the fact that good institutions make the economic environment more efficient. However, some practices undermine the strength and effectiveness of institutions: corruption, lack of law, inconsistency of some rules, incompetence, etc. In these situations, weak or poor institutions create uncertainty, bad governance and lead to low growth.

Despite the multitude of studies available up to now, the relevant literature shows no consensus on way to conceptualize the institutions' quality or the occurrence of institutional change. Several of them mention the important role of a political actor (State or Government) in implementing efficient institutions [Zak (2000), Leukert (2005), Francois (2006)]. However, the role of the State is specifically contextualized. In developing countries with a quasi-absence of private saving, the government is the largest investor in the economy. Then, both production and labor markets are controlled by the government. What if the government fails to build efficient institutions? Our goal is to build a theoretical framework of institutional change within a developing country where governmental corruption is a source of institutional weakness.

Economic theory has developed two basic views of corruption respectively exogenous and endogenous corruption to the political process. The first type is modelled following the principle-agent approach based on the assumption of the existence of an asymmetry of information between principals (politicians or decision makers) and agents (civil servants); the second type means institutionalized corruption since it involves the political regime. Analyzing corruption as an endogenous phenomena is more realistic, but up to now it has not provided a clear analytical framework for the level and structure of corruption (Begovic, 2005). The contribution of this investigation is an attempt to offer a way to conceptualize an analytical framework for endogenous corruption in the light of New Institutional Economics [North (1990), Rodrik (1999)], which considers institutions as endogenous phenomena.

We explore this issue by modelling the role of an active civil society in building good institutions. As indicated in the literature, Civil societies are often populated by organizations such as registered charities, non-governmental development organizations, community groups, women's organizations, faith-based organizations, professional associations, trades unions, self-help groups, social movements, business associations, coalitions and ad-

vocacy groups². Summarizing the key elements describing civil society, the Association for Research and Promotion of Participative Democracy in Eastern Europe (AREDA) stipulates that: " The Civil Society is the totality of groups (NGOs) and individuals in a country, who show a regular concern for the social and political context in that country, without fulfilling the function of political parties, who are autonomous from the government, and to whose goals also belongs to monitor the activity of the government or certain specific consequences of it, as well as to resist - if possible peacefully - any unlawful, dangerous or abusive government activity" (AREDA, 2003).

Skidmore (2001) argues that civil society in developing countries may play an important role in promoting good governance since it helps to build effective and accountable States and supports voices for changes and that even consumers sometimes join or contribute to organizations that advocate their interests.

In reality, the appearance and proliferation of civil society organizations throughout the world can be seen as a request for efficient institutions addressed to public authorities. Accordingly, civil society can be interpreted as a sphere that evolves largely in response to the inability or failure of the State to meet the needs or demands of its citizens (Hyden, 1997). Ordinary citizens may then combine their efforts to build strong civil society as a coalition of organizations who react strongly to anti-productive Governmental activities either, in Hirschman's (1970) terms, by voice or by loyalty. According to Hirschman (1970)³ and Dowding et al (2000), civil society can either protest (voice) against a corrupt Government, stay silent (loyal) or exit. Hirschman (1970) defines the voice option as

" Any attempt at all to change, rather than to escape from, an objectionable state of affairs, whether through individual or collective petition to the management directly in charge, through appeal to higher authority with the intention of forcing a change in management, or through various types of actions and protests, including those that are meant to mobilize public opinion".

In this paper, the exit option is avoided since we want to model the role of active civil society in fighting corruption. Civil society still has two strategies: voice or loyalty. On its side, Government can repress or tolerate any attempt

²This characterization is commonly used by the World Bank (<http://www.worldbank.org>), International Monetary Found (<http://www.imf.org>), Centre for Civil Society at London School of Economics (<http://www.lse.ac.uk/collection/CCS>), etc

³Hirschman suggests there are two consumer reactions to a decline in product quality: exit and voice, which interact with a third, loyalty.

at revolt by handling its mechanisms of repression: the police force, prison, courts of justice and administrative constraints such as the prohibition of an association within national territory. AREDA adds that governmental hostility can lead Governments to create and finance an artificial civil society in order to deter the pressure of active organizations. Authoritarian governments adopt repressive strategies to reinforce its capacity to create rents; less bad governments act passively by not employing punishment mechanisms against the protests of civil society.

Below, we explore actual civil society monitoring activities, using the following historical examples of effective actions across the World:

1. From the OECD experience, we learn that various civil society organizations such as BIAC, TUAC, ICC and TI⁴ supported the process which led to the adoption of the Convention on Combating Bribery of Foreign Public Officials in International Business Transactions in 1999. Civil society organizations provide useful links between public actors and the business community and contribute to maintaining pressure on governments to fulfil their commitments (Hors, 2003)
2. The Initiative Policy Dialogue founded in July 2000 by the Nobel laureate economist Joseph Stiglitz is a collaborative effort among nearly 200 leading economists, political scientists, policymakers and civil society representatives from developed and developing countries. Its efforts are intended to help countries find solutions to pressing problems and also to strengthen their institutions and civil societies (Columbia University, www.gsd.columbia.edu)
3. Through the Strengthening Civil Society Monitoring Capacity in Moldova (SCSMCM) program, IREX (International Research and Exchanges Board) supports journalists, media outlets and media non-governmental organizations (NGOs) in their "watch-dog" role as the Moldovan government undertakes reforms to reduce corruption as part of its commitment to the Millennium Challenge Corporation Country Threshold Program. (www.irex.org)
4. Civil society in Uganda has used a combination of approaches to the fight against corruption. These include: formation by 40 Civil society organizations of an Anti-Corruption Coalition (ACCU) to or-

⁴BIAC, TUAC, ICC and TI respectively stand for Business and Industry Advisory Committee, Trade Union Advisory Committee, International Chamber of Commerce and Transparency International

ganise jointly the fight against corruption (ACCU organizes an anti-corruption week in October countrywide); promoting public education and the mobilization of people to demand the accountability of public officials and to resist corruption at all levels (Uganda Debt Networks is an example of a CSO working in this domain); establishment of a Centre for Corporate Governance to ensure ethical behaviour to corporations in their business dealings; involvement of CSOs in monitoring the Poverty Action Fund (PAF) through which the funds from debt relief and other donors are channelled to eradicate poverty; coordination between CSOs and Government for effectiveness in the fight against corruption (Gariyo, 2001).

5. In Bulgaria, administrative corruption was slashed to half its 1998 level after the Center for the Study of Democracy (CSD) and the Coalition 2000 it founded managed to bring anti-corruption efforts to the forefront of the policy agenda and to make the public less tolerant of rent seeking by public officials (Islam, 2008).

These examples illustrate the monitoring role of civil society in installing good governance by combating corruption. Consequently, we construct a theoretical dynamic game between government and civil society to explore their interaction in the economy. We distinguish between competitive play and cooperative arrangement. Three regimes are considered: Regime 1 and Regime 2 reflect the competitive play, while Regime 3 is the cooperative mood. In Regime 1, active or loyal civil society interacts with passive government; Regime 2 invokes the dynamic conflict between active civil society and an oppressive government.

The rest of this paper is organized as follows: Section 2 is devoted to the model of competitive interaction. In section 3, we present the model of cooperative arrangement and in Section 4, we assess numerically the robustness of theoretical results before concluding in Section 5.

2 The model of competitive interaction

Let us consider a developing country with substantial public involvement in the production sector. The private sector is not sufficiently developed to dominate the economy. Hence, the quality of institutions depends on government behavior. Government owns the firms owner. The economy is populated by a continuum of identical consumers who inelastically supply

labor to produce output according a standard production function

$$Y_t = A_t F(L_t)$$

such that $A_t > 0$; $F' > 0$, $F'' < 0$. A_t is the total factor productivity (TFP) through which institutions enter the production sector as an externality. We assume that the economic environment, characterized by the prevailing institutional structure of the economy, exerts a positive indirect effect on output. Formally, $\frac{\partial Y_t}{\partial A_t} = F(L_t) > 0$ and $A_I = \frac{\partial A_t}{\partial I_t} > 0$ where I_t is the institutional quality index of the economy⁵.

Following Leukert(2005)⁶, we assume that institutional quality I_t evolves over time according to the following law of motion

$$\dot{I}_t = bw_t - \delta I_t \tag{1}$$

where δ is the depreciation rate. This indicates that if consumers do not care about improving institutions, they deteriorate at rate δ . The parameter b captures the efficiency of civil monitoring activities.

Consumers' participation in civil society implies that some labor is used outside the production sector. Assume the time available to each consumer is normalized to one. Then, the time constraint for consumers is $L_t + w_t = 1$

For any date t , it is assumed that a proportion ϕ of total production is stolen by the government. The function ϕ is endogenous and depends negatively on the effort w_t devoted to improving institutions by civil society (consumers) and positively on the pressure x_t Government exerts on civil society. In the other words, the function ϕ shows the corruption technology of the government and $0 < \phi < 1$.

The level of consumption at time t is then $C_t = (1 - \phi(w_t, x_t))Y_t$ and the production function becomes

$$Y_t = A_t F(L_t) \text{ or } Y_t = A_t F(1 - w_t)$$

⁵The total factor of productivity $A_t = A(I_t, z_t)$ where I_t is the institutional quality and z_t a vector of other factors

⁶Leukert(2005) indicates that agents decide how much effort to invest in improving the informal institutions at each point in time. The change of formal institutions is the job of the State whose aim is to instill the good habits. In our case, Government is corrupt and we assume that the improvement of institutions will happen through the existence of an active civil society through which each consumer acts in order to constrain the government to build efficient institutions.

This set up implies that civil monitoring has two opposite effects on output: an indirect positive effect of w on y (TFP effects through institutional improvement) and a detrimental direct effect of w on y since an increase in w reduces the time devoted to production activities. The total effect depends on which impact dominates.

We begin by analyzing the case where civil society can protest (voice) or can be loyal in the presence of a non-authoritarian corrupt government. This case refers to a one-player differential game or a simple optimal control problem.

2.1 Active or loyal civil society versus passive government

In this Regime 1, corruption technology depends only on civil monitoring w : $\phi_t = \phi(w_t)$. A representative consumer's dynamic problem with discounting at rate ρ can be formulated as

$$\text{Max}_{w_t} \int_0^{\infty} \exp(-\rho t) U(C_t) dt, \text{ subject to Equation (1)}$$

with $0 \leq w_t \leq 1$. This relation leads to two inequality constraints: $g_1 \equiv w \geq 0$ and $g_2 \equiv 1 - w \geq 0$

The corresponding current value of the Hamiltonian is given by:

$$H(w_t, I_t, \lambda_t) = U[(1 - \phi(w_t))A_t F(1 - w_t)] + \lambda_t [bw_t - \delta I_t]$$

where λ_t denotes the co-state variable associated to (1).

The optimality conditions consist of the law of motion of institutional quality index (1) and

$$-U_c \phi'(w)Y - U_c(1 - \phi(w))AF'_L + \lambda b = 0 \quad (2)$$

$$(1 - \phi(w))U_c F(\tilde{K}, 1 - w)A_I - \delta \lambda = \rho \lambda - \dot{\lambda} \quad (3)$$

$$\lim_{t \rightarrow \infty} \lambda_t I_t = 0. \quad (4)$$

Equation (4) gives the transversality condition. Since the problem above is an inequality-constraint problem, we need to invoke the Kuhn-Tucker necessary conditions. Thus, the current-value Hamiltonian is augmented into a Lagrangian function as follows

$$L(w_t, I_t, \lambda_t, \mu_{1t}, \mu_{2t}) = H + \mu_{1t}w_t + \mu_{2t}(1 - w_t)$$

First order conditions (FOCs), known as Kuhn-Tucker conditions for maximizing L , are given by⁷

$$L_w \equiv -U_c\phi'(w)Y - U_c(1 - \phi(w))AF'_L + \lambda b + \mu_1 - \mu_2 = 0 \quad (5)$$

$$L_{\mu_1} \equiv w \geq 0; \mu_1 \geq 0; \mu_1 w = 0 \quad (6)$$

$$L_{\mu_2} \equiv (1 - w) \geq 0; \mu_2 \geq 0; \mu_2(1 - w) = 0 \quad (7)$$

$$L_I \equiv \rho\lambda - \dot{\lambda} = (1 - \phi(w))U_c F(1 - w)A_I - \delta\lambda \quad (8)$$

$$L_\lambda \equiv \dot{I} = bw - \delta I \quad (9)$$

Following the Mangasarian sufficiency theorem, the necessary conditions of the maximum principle (5) - (9), are also sufficient for global maximization iff $\phi_{ww} \leq 0$ and $A_{II} \leq 0$ (See appendix A for more details).

Analytical solutions require explicit functions to be chosen. Taking an economy with linear production function $Y_t = A(1 - w_t)$; linear utility function $U(C_t) = C_t$, and linear corruption technology $\phi(w_t) = \kappa(1 - w_t)$ with $0 < \kappa < 1$, where the parameter κ can be defined as the capacity of government to create rent, we can see that the consumption function is quadratic in w and the state equation is linear in w . We have refined the popular Linear Quadratic Differential Game. Total factor productivity is modelled as a function of the institutional quality and other variables z_t such that there exists positive interaction between the z variables and governance quality in the economy: $A_t = z_t(A_0 + I_t)$ for $A_0 > 0$. These functional forms help analytical tractability.

If civil society chooses to voice i.e $w \neq 0$, the FOCs can be solved technically for an interior solution. An interior solution $w^* \in]0; 1[$ exists iff the inequality constraints $g_i(w^*)$ are not binding: $g_i(w^*) > 0 \forall i = 1, 2$ and complementary-slackness conditions given by $\mu_1 w = 0$ and $\mu_2(1 - w) = 0$, respectively in conditions (6) and (7) imply that $\mu_1 = \mu_2 = 0$. Solving (5) - (9) for an interior optimal solution, given the explicit functions above, we find the following system

$$(2\kappa(1 - w) - 1)A + \lambda b = 0 \quad (10)$$

$$\dot{\lambda} = (\delta + \rho)\lambda - (1 - \kappa(1 - w))(1 - w)z \quad (11)$$

$$\dot{I} = bw - \delta I \quad (12)$$

⁷For simplicity, time subscripts are dropped

Proposition 1 (Existence of the equilibrium): *There exists at least one solution $0 < w^* < 1$ for A_0 small enough.*

Proof: The Euler equation (10) gives us the effort-setting equation and can be rewritten as

$$2\kappa z(A_0 + I)w - \lambda b - (2\kappa - 1)z(A_0 + I) = 0 \quad (13)$$

Let us denote the Left Hand Side of (13) by $F(w)$ and apply the Intermediate Values Theorem to the interval $[0; 1]$. By definition, if F is a continuous function on the interval $[0; 1]$ and if the condition $F(0)F(1) \leq 0$ is verified, then the equation $F(w) = 0$ has at least one solution on the same interval. From the dynamic system of Equations (11) and (12), we have the following steady state values

$$\lambda^* = \frac{(1 - \kappa(1 - w^*))(1 - w^*)z}{(\delta + \rho)} \quad (14)$$

$$I^* = \frac{b}{\delta}w^* \quad (15)$$

Using equations (13) - (15) we can easily show that $F(1) = z(A_0 + \frac{b}{\delta}) > 0$ and $F(0) = -\lambda b = -\frac{(1 - \kappa)z}{(\delta + \rho)} - (2\kappa - 1)zA_0$. For A_0 small enough, $F(0) = -\frac{(1 - \kappa)z}{(\delta + \rho)}$. Thus, $F(0)F(1) < 0$. ■

Proposition 2 (Uniqueness and stability of the equilibrium):

The system of Equations (10)-(12) admits a unique stable saddlepoint equilibrium (w^, I^*, λ^*) in the space (I, λ) .*

Proof :

By plugging Equations (14) and (15) into Equation (13) and rearranging, we obtain the following optimal civil monitoring:

$$w^* = \frac{-2A_0\kappa\delta + b(2\kappa - 1)(2\delta + \rho) + \Lambda}{2b\kappa(3\delta + 2)} \quad (16)$$

Where:

$$\Lambda = \sqrt{-4b\kappa\delta(3\delta + 2\rho)(b(\kappa - 1) - A_0(2\kappa - 1)(\delta + \rho)) + (-2A_0\delta\kappa + b(2\kappa - 1)(2\delta + \rho))^2}$$

Plugging Equation (16) into Equations (15) and (14) gives the steady states values I^* and λ^* respectively in terms of the model's parameters.

Let us now study the stability of this equilibrium. The dynamic system of Equations (11) - (12) can be rewritten as

$$\dot{I} = b \frac{(2\kappa - 1)A + \lambda b}{2A\kappa} - \delta I \quad (17)$$

$$\dot{\lambda} = (\delta + \rho)\lambda - (1 - \kappa(1 - w))(1 - w)z \quad (18)$$

The associated Jacobian matrix evaluated at the steady state is given by

$$\begin{pmatrix} -\frac{\lambda^* b^2}{2\kappa z (A_0 + I^*)^2} - \delta & \frac{b^2}{2\kappa z (A_0 + I^*)} \\ 0 & (\delta + \rho) \end{pmatrix}$$

Since this Jacobian matrix is triangular, the two eigenvalues are $(\delta + \rho)$ and $-\frac{\lambda^* b^2}{2\kappa z (A_0 + I^*)^2} - \delta$, which have opposite signs. Then the interior solution (w^*, I^*, λ^*) is a local stable saddlepoint. ■

The dynamic optimization problem above should admit two corner solutions $w^* = 0$ and $w^* = 1$. However $w^* = 1$ is excluded by construction since production would be equal to zero and thus so would consumption. Let us explore the only possible case $w^* = 0$. If the model admits the corner solution of zero optimal effort, this implies that civil society is loyal i.e does nothing in fight against corruption. Under this hypothesis, the inequality constraint $g_1(w)$ is binding i.e $g_1(w^*) = 0$ with $w = 0, \mu_1 > 0$ and $\mu_1 w = 0$

Proposition 3 (Existence and stability of the corner solution):

The corner regime displays one stable saddlepoint solution of zero optimal effort, iff $\kappa < \frac{A_0(\delta + \rho) - b}{2A_0(\delta + \rho) - b}$. For A_0 small enough, the corner solution exists and is stable for any $\kappa \in]0; 1[$

Proof:

According to the Khun-Tucker conditions (5) - (9), after imposing $w^* = 0$ and $\mu_2 = 0$, the Euler equation (10) becomes

$$(2\kappa - 1)zA_0 + \lambda b + \mu_1 = 0 \quad (19)$$

Since $w^* = I^* = 0$, the steady state value is

$$\lambda^* = \frac{(1 - \kappa)z}{(\delta + \rho)} \quad (20)$$

Plugging Equation (20) into Equation (19) gives

$$\mu_1^* = (1 - 2\kappa)zA_0 - b \frac{(1 - \kappa)z}{(\delta + \rho)} \quad (21)$$

From Equation (21), it follows that $\mu_1^* > 0$ iff $\kappa < \frac{A_0(\delta + \rho) - b}{2A_0(\delta + \rho) - b}$.

Furthermore, the Jacobian matrix associated with this corner solution can be computed as follows

$$\begin{pmatrix} -\frac{(1 - \kappa)b^2}{2\kappa z A_0^2} - \delta & \frac{b^2}{2\kappa z A_0} \\ 0 & (\delta + \rho) \end{pmatrix}$$

This Jacobian matrix shows that the two eigenvalues $-\frac{(1 - \kappa)b^2}{2\kappa z A_0^2} - \delta$ and $(\delta + \rho)$ have opposite signs. Thus, the corner stationary solution of zero optimal effort is a stable saddlepoint. ■

2.2 Active civil society versus oppressive government

We can construct a theoretical Nash game in which active civil society faces an authoritarian government. Consider a two-agent differential game in which time is continuous, the game is played on the infinite horizon. Recall that the government seeks, per unit of output Y_t , current rent $\phi_t = \phi_t(x, w)$ depending negatively on the effort w and positively on the pressure x it exerts on civil society: $\phi_x > 0$ and $\phi_w < 0$ as in the previous case. However, the government incurs the cost of implementing and enforcing sanctioning mechanisms. Let $g(x)$ denotes the cost function which is increasing and convex: $g_x > 0$ and $g_{xx} \geq 0$. As previously, a representative consumer receives the amount $(1 - \phi(x, w))$ per unit of output Y_t .

Government's objective is to maximize the present value of its benefits from corruption minus the expenses devoted to punishment mechanisms. We assume that the discount factor $\rho > 0$ is the same for both agents. Then, the two dynamic optimization problems are:

For government:

$$\text{Max}_{x_t} \int_0^\infty \exp(-\rho t) [U(G_t) - g(x_t)] dt \text{ subject to Equation (1)}$$

where G_t is the total amount of corruption and $G_t = \phi(x_t, w_t)Y_t$.

For a representative consumer:

$Max_{w_t} \int_0^\infty exp(-\rho t)U(C_t)dt$ subject to Equation (1)

This is the same as Regime 1, except that corruption ϕ depends also on governmental hostility x .

The current value Hamiltonian associated with the Government's optimization problem is

$$H(x_t, I_t, \eta_t) = U(\phi(x_t, w_t)Y_t) - g(x_t) + \eta(bw_t - \delta I_t)$$

where η is the costate variable

The necessary optimality conditions are given by:

$$H_x \equiv U_G \phi_x Y - g_x = 0 \tag{22}$$

$$H_I \equiv \rho \eta - \dot{\eta} = U_G \phi A_I F(L) - \delta \eta \tag{23}$$

$$H_\eta \equiv \dot{I} = bw_t - \delta I \tag{24}$$

The Mangasarian sufficiency theorem for the maximum principle (22) - (24) holds iff $\phi_{xx} \leq 0$ (see Appendix B). The method of resolution of the consumer's problem is identical to the preceding case.

Definition 1: A pair of strategies (\bar{x}, \bar{w}) is an open-loop Nash equilibrium if the two strategies simultaneously maximize the dynamic optimization problems above. In other words, if the necessary optimality conditions of the optimal control problems hold simultaneously.

To get the open-loop Nash equilibrium, it is enough to solve the system of Equations (5) and (22). We consider the following quadratic preferences for government:

$$U(G_t) = \alpha G_t - \frac{\beta}{2} G_t^2 \tag{25}$$

where α and β are real numbers and β captures the government's aversion to rent variations. Furthermore, we restrict ourselves to the convex cost function for implementing punishment mechanisms and a linear corruption technology ϕ on both x and w . Thus, we consider:

$$g(x_t) = \frac{x_t^2}{2} \tag{26}$$

$$\phi(x_t, w_t) = \kappa(1 - w_t + x_t) \tag{27}$$

As in the previous case, we take a linear utility function for consumers .

Solving the Equations (5) and (22) respectively for x and w yields

$$x = \frac{\alpha\kappa A(1-w) - \beta\kappa^2 A^2(1-w)^3}{1 + \beta\kappa^2 A^2(1-w)^2} \quad (28)$$

$$w = \frac{(2\kappa - 1 + \kappa x)A + \lambda b}{2A\kappa} \quad (29)$$

From Equation (28), we can easily deduce that x is positive iff $\beta < \frac{\alpha}{\kappa A(1-w)^2}$. Plugging Equation (28) into Equation (29) and solving for w , we obtain the following sixth-order polynomial in w :

$$A\kappa\{\beta\kappa A(1-w)^2[\kappa A(1-w) - (\lambda b + (2\kappa - 1)A)] + 2w(1 + \beta\kappa^2 A^2(1-w)^2) - \alpha\kappa A(1-w)\} - \lambda b - (2\kappa - 1)A = 0 \quad (30)$$

Proposition 4: *There exists at least one Open-loop Nash equilibrium (\bar{w}, \bar{x}) such that $0 < \bar{w} < 1$ for A_0 small enough. Furthermore, for β small enough this solution is unique and displays a stable saddle point.*

Proof: Let us denote by $P(w)$ the Left Hand Side (LHS) of the Equation (30). Using the Intermediate Values Theorem, we can easily show that $P(0)P(1) < 0$ which ensures the existence of at least one solution $0 < \bar{w} < 1$.

Indeed, computing all the FOCs of the two problems above yields the following dynamical system:

$$\dot{I} = b\left(\frac{(2\kappa - 1 + \kappa x)A + \lambda b}{2A\kappa}\right) - \delta I \quad (31)$$

$$\dot{\lambda} = (\delta + \rho)\lambda - z(1 - \kappa(1 - w + x))(1 - w) \quad (32)$$

$$\dot{\eta} = (\delta + \rho)\eta + \beta\phi^2 z^2 (A_0 + I)(1 - w)^2 - \alpha\phi z(1 - w) \quad (33)$$

At steady state, $\dot{\lambda} = \dot{\eta} = \dot{I} = 0$ implying that

$$\bar{\eta} = \frac{(\alpha - \beta\phi z(A_0 + \bar{I})^2(1 - \bar{w}))\phi z(1 - \bar{w})}{(\delta + \rho)} \quad (34)$$

$$\bar{I} = \frac{b}{\delta}\bar{w} \quad (35)$$

$$\bar{\lambda} = \frac{z(1 - \kappa(1 - \bar{w} + \bar{x}))(1 - \bar{w})}{(\delta + \rho)} \quad (36)$$

On the one hand, setting $w = 1$ implies that $\bar{x} = \bar{\lambda} = 0$ and $\bar{I} = \frac{b}{\delta}$. On the other hand, if $w = 0$; $\bar{I} = 0$ and

$$\bar{x} = \frac{\alpha\kappa z A_0 - \beta\kappa^2 z^2 A_0^2}{1 + \kappa^2 z^2 A_0^2}$$

$$\bar{\lambda} = \frac{(1 - \kappa(1 + \frac{\alpha\kappa z A_0 - \beta\kappa^2 z^2 A_0^2}{1 + \kappa^2 z^2 A_0^2}))z}{(\delta + \rho)}.$$

Note that for A_0 small enough, $\bar{x} = 0$ and $\bar{\lambda} = \frac{(1-\kappa)z}{(\delta+\rho)}$. Computing $P(1)$ and $P(0)$ we find that $P(1) = A = z(A_0 + \frac{b}{\delta}) > 0$ and $P(0) = -b\frac{(1-\kappa)z}{(\delta+\rho)} < 0$ implying that $P(0)P(1) < 0$.

Let us now study the uniqueness and stability of the equilibrium. For β small enough, the LHS of Equation (30) is reduced to the following third-order polynomial in w :

$$2A\kappa w - \alpha\kappa^2 A^2(1 - w) - \lambda b - (2\kappa - 1)A = 0 \quad (37)$$

After plugging Equation (36) into equation (37), and replacing the TFP by its expression and providing some transformations, the Polynomial (37) can be rewritten as follows:

$$\frac{\alpha z \kappa^2}{\delta} ((\delta + \rho)\frac{b}{\delta} + 1)w^3 + [2\kappa(\delta + \rho)\frac{b}{\delta} + \kappa - \alpha z \kappa^2 ((\delta + \rho)\frac{b^2}{\delta^2} + \frac{2b}{\delta})]w^2 + [(1 - 2\kappa)(\delta + \rho)\frac{b}{\delta} + (1 - 2\kappa) + \alpha z \kappa^2 \frac{b}{\delta}]w + \kappa - 1 = 0$$

This third-order polynomial displays three solutions, two of which are complex and one is a real root which is not easily interpretable. For β and A_0 small enough, the jacobian matrix $J(I^*, \lambda^*, \eta^*)$ associated with the dynamic system (31) - (33) is

$$\begin{pmatrix} -\delta - \frac{\lambda^* \kappa z}{2} & \frac{b^2}{2\kappa z(A_0 + I^*)} & 0 \\ 0 & (\delta + \rho) & 0 \\ 0 & 0 & (\delta + \rho) \end{pmatrix}$$

The Jacobian matrix evaluated at the steady state is a triangular matrix with one negative eigenvalue and two positive ones, $\nu_1 = -\delta - \frac{\lambda^* \kappa z}{2}$ and $\nu_2 = \nu_3 = (\delta + \rho)$. We conclude that the dimension of the stable manifold i.e the number of eigenvalues with negative real parts is equal to one. Thus, the unique Open-loop Nash equilibrium is a stable saddle point. ■

3 Cooperative arrangement

Let us now consider the case when civil society and government agree to cooperate. Under a cooperative arrangement players seek a set of strategies

that ensure a Pareto optimal solution. The objective function to be maximized is the weighted sum of the two payoffs:

$$\max_{w,x} [\int_0^\infty \exp(-\rho t) \{ \pi [U(G_t) - g(x_t)] + (1-\pi)U(C_t) \} dt \text{ subject to (1)} \\ \text{with } 0 < \pi < 1$$

where π is the cooperation weight of the government. The augmented lagrangian function can be written as:

$$L = \pi [U(G_t) - g(x_t)] + (1-\pi)U(C_t) + \lambda_t (bw_t - \delta I_t) + \mu_{1t} w_t + \mu_{1t} (1-w_t) \quad (38)$$

Computing the FOCs for interior solutions yields

$$A[-1 + \pi + (-2 + 2w - x)\gamma_1 - \gamma_2(-1 + w)(-1 + w - x)(2(-1 + w) - x)] + \lambda b = 0 \quad (39)$$

$$-x + \psi_1(-1 + w) + \psi_2(1 - w)^2(1 - w + x) = 0 \quad (40)$$

$$\dot{\lambda} = \theta_1(-1 + w)^2(1 - w + x)^2 - (-1 + w)z(-1 + \pi + (-1 + w - x)\theta_2) + (\delta + \rho)\lambda \quad (41)$$

$$\dot{I} = bw - \delta I \quad (42)$$

with

$$\begin{aligned} \gamma_1 &= (-1 + \pi(1 + \alpha))\kappa \text{ and } \gamma_2 = A\pi\beta\kappa^2 \\ \psi_1 &= A\alpha\kappa \text{ and } \psi_2 = A^2\beta\kappa^2 \\ \theta_1 &= \pi A\beta z(\kappa)^2 \text{ and } \theta_2 = \gamma_2 = (-1 + \pi(1 + \alpha))\kappa \end{aligned}$$

Definition 2: (\bar{w}, \bar{x}) is a pair of pareto strategy if it solves the system of Equations (39)-(40) given the dynamic Equations (41) and (42).

Proposition 5: For A_0 small enough and $\pi < \frac{1}{\alpha+1}$, there exists at least one solution $0 < w^* < 1$

Proof: Solving Equation (40) for x , we get

$$x = \frac{A(-1 + w)\kappa(1 - \pi(1 + \alpha) + A\pi(-1 + w)^2\beta\kappa)}{\pi(1 + A^2(-1 + w)^2\beta\kappa^2)} \quad (43)$$

Plugging Equation (43) into Equation (39), we obtain an eight-order polynomial $P(w) = 0$. We can study the existence of $w^* \in [0, 1]$ by applying the Intermediate Values Theorem to this function. Knowing that at steady state

$$\begin{aligned} I^* &= \frac{b}{\delta} w^* \text{ and} \\ \lambda^* &= \frac{-\pi(-1+w)^2(1-w+x)^2(A_0+I)(z\kappa)^2\beta + (-1+w)z(-1+\pi+(-1+w-x)(-1+\pi(1+\alpha))\kappa)}{(\delta+\rho)}, \end{aligned}$$

we can easily show that $P(1) = -z(A_0 + \frac{b}{\delta})(1 - \pi) < 0$. Furthermore, for A_0 small enough $P(0) = -\frac{zb(1 - \kappa + \pi(-1 + \kappa + \alpha\kappa))}{(\delta + \rho)} > 0$ iff $\kappa \leq \frac{1 - \pi}{1 - \pi(\alpha + 1)}$. Since $0 < \kappa < 1$, the last condition requires that $\pi < \frac{1}{\alpha + 1}$. Thus, under the condition $\pi < \frac{1}{\alpha + 1}$; $P(0) > 0$ and by the Intermediate Values Theorem, $P(0)P(1) < 0$, implying that there exists at least one solution $0 < w^* < 1$. ■

Proposition 6: *For β small enough, the Pareto strategy (w^*, x^*) is uniquely determined and the associated stationary solution is a stable saddle point.*

Proof:

Imposing β and A_0 small enough, the eight-order polynomial becomes the third-order polynomial in w

$$P(w) = \varphi_1 w^3 + \varphi_2 w^2 + \varphi_3 w + \varphi_4 \quad (44)$$

where

$$\begin{aligned} \varphi_1 &= \frac{b\kappa(\pi(1 + \alpha) - 1)[\delta^3\pi + b\kappa(\pi(1 + \alpha) - 1)]}{\delta^2\pi} \\ \varphi_2 &= \frac{2b\kappa(\pi(1 + \alpha) - 1)[1 - \delta^2] + \pi\delta^3}{\delta} \\ \varphi_3 &= \frac{\delta\pi[b(\pi - 1)(2\delta + \rho) - 2\delta^2\pi(\delta + \rho)] + b\kappa(\pi(1 + \alpha) - 1)[(\delta + \rho)\delta^3\pi - b\kappa(\pi(1 + \alpha) - 1)]}{(\delta + \rho)\delta^3\pi} \\ \varphi_4 &= \frac{\pi\delta^3(\delta + \rho) - b(\pi - 1)\delta - 2b\kappa(\delta + \rho)(\pi(1 + \alpha) - 1)}{\delta(\delta + \rho)} \end{aligned}$$

Solving Equation (44) for w , we obtain one real solution and two complex ones. The complex solutions have to be excluded but the analytical form of the real solution is not easily interpretable.

Let us now study the stability of the unique cooperative outcome. The Jacobian matrix associated to the dynamic system in Equations (41)-(42) and evaluated at steady state can be written as

$$\begin{pmatrix} -\delta + J_{11} & \frac{-b}{2\kappa z(A_0 + \bar{I})(-1 + \pi(1 + \alpha))} \\ 0 & (\delta + \rho) \end{pmatrix}$$

where $J_{11} = \frac{-(\bar{w} - 1)z(-1 + \pi(1 + \alpha))\kappa}{2\pi} + \frac{b\bar{\lambda}}{2z(A_0 + \bar{I})^2(-1 + \pi(1 + \alpha))\kappa} < 0$

This triangular matrix has one negative eigenvalue $J_{11} - \delta$ and one positive eigenvalue $(\delta + \rho)$. Thus, the pair of Pareto strategy (w^*, x^*) is a stable saddle point. ■

4 Robustness numerical assessment

We assess numerically the analytical results obtained in the above three regime (namely the simple optimal control when an active civil society faces a passive government, the open-loop Nash equilibrium characterizing the outcome of the interaction between active civil society and oppressive government, and the cooperative outcome).

4.1 Calibration

The parameters of the model are calibrated according to the conditions to be met for the existence and uniqueness of the equilibrium in all three regimes.

Table 1: Calibration of parameters

| Parameter | δ | b | A_0 | ρ | α | β | π | z |
|-----------|----------|-----|-------|--------|----------|---------|-------|-----|
| Value | 0.1 | 0.5 | 0.001 | 0.98 | 0.5 | 0.002 | 0.6 | 1 |

4.2 Sensitivity analysis

Below we consider two examples: a variation of κ and a variation of β , other factors remaining unchanged.

For different values of κ such that $0 < \kappa < 1$ with others factors unchanged, it can be shown graphically that there exists a unique optimal effort $w^* \in]0, 1[$ and this increases as κ increases. (see Figures 1 and 2). The impact of an increase of κ on civil monitoring and output is computed for the three regimes. As a corollary, the speed of convergence to the long run equilibrium which is the modulus of the negative (stable) eigenvalue is computed. Recall that in the three regimes, the negative eigenvalue is respectively given by:

$$v_1 = -\frac{\lambda^* b^2}{2\kappa z (A_0 + I^*)^2} - \delta$$

$$v_2 = -\delta - \frac{\lambda^* \kappa z}{2}$$

$$\text{and } v_3 = -\delta + J_{11}$$

$$\text{where } J_{11} = \frac{-(\bar{w}-1)z(-1+\pi(1+\alpha))\kappa}{2\pi} + \frac{b\bar{\lambda}}{2z(A_0+I)^2(-1+\pi(1+\alpha))\kappa} < 0$$

Table 2: Active civil society versus passive government (Regime 1)

| κ | w^* | I^* | λ^* | y^* | v_1 |
|----------|-------|-------|-------------|-------|---------|
| 0.05 | 0.088 | 0.442 | 0.806 | 0.404 | -10.353 |
| 0.1 | 0.093 | 0.465 | 0.764 | 0.423 | -4.488 |
| 0.2 | 0.106 | 0.528 | 0.680 | 0.473 | -1.617 |
| 0.4 | 0.158 | 0.791 | 0.517 | 0.666 | -0.730 |
| 0.5 | 0.210 | 1.051 | 0.442 | 0.831 | -0.200 |
| 0.7 | 0.352 | 1.761 | 0.328 | 1.141 | -0.119 |
| 0.75 | 0.386 | 1.931 | 0.307 | 1.186 | -0.114 |
| 0.9 | 0.474 | 2.372 | 0.256 | 1.247 | -0.106 |

Table 2 the following economic implications: as κ increases, both the optimal effort and the optimal output also increase. Then, given our calibration above, the indirect positive effect of w on y (TFP effects) always dominates the negative direct effect.

Table 3: Active civil society versus oppressive government (Regime 2)

| κ | w^* | I^* | λ^* | y^* | v_2 |
|----------|-------|-------|-------------|-------|---------|
| 0.05 | 0.088 | 0.442 | 0.805 | 0.404 | -10.348 |
| 0.1 | 0.093 | 0.465 | 0.762 | 0.423 | -4.476 |
| 0.2 | 0.106 | 0.530 | 0.672 | 0.475 | -1.590 |
| 0.4 | 0.168 | 0.843 | 0.471 | 0.702 | -0.306 |
| 0.5 | 0.256 | 1.279 | 0.351 | 0.953 | -0.153 |
| 0.7 | 0.525 | 2.625 | 0.159 | 1.247 | -0.104 |
| 0.75 | 0.577 | 2.885 | 0.133 | 1.221 | -0.103 |
| 0.9 | 0.691 | 3.454 | 0.083 | 1.068 | -0.101 |

The steady state values given in Table 3 illustrates that as κ increases, civil monitoring will always increase. However, there exists a threshold value κ^* at which the indirect positive effect of institutional improvement (TFP effects) and direct detrimental effect of civil monitoring on output totally compensate. For all $\kappa < \kappa^*$ optimal output will always increase, meaning that positive TFP effects dominate the negative civil monitoring effect. For all $\kappa > \kappa^*$ the negative civil monitoring effect dominates and output will always decrease with κ . After several iterations $\kappa^* = 0.70$ (see Figure 3).

Table 4: Cooperative play (Regime 3)

| κ | w^* | I^* | λ^* | y^* | v_3 |
|----------|-------|-------|-------------|-------|---------|
| 0.05 | 0.085 | 0.427 | 0.333 | 0.392 | -90.935 |
| 0.1 | 0.086 | 0.432 | 0.294 | 0.324 | -43.359 |
| 0.2 | 0.088 | 0.442 | 0.274 | 0.297 | -19.024 |
| 0.4 | 0.093 | 0.465 | 0.269 | 0.200 | -5.880 |
| 0.45 | 0.094 | 0.472 | 0.252 | 0.166 | -4.261 |

In Regime 3, the optimal effort always increases with κ while the optimal output always decreases with κ . This implies that indirect positive effect (TFP effect) is dominated by the negative direct effect of w on y . Furthermore, as κ approaches 0.5, the cooperation scenario is not profitable to consumers since the amount of corruption per unit of production approaches one and consumption tends to zero. This analysis is therefore restricted to cases where $\kappa < 0.5$.

In summary, the numerical assessment indicates that $\frac{\Delta w^*}{\Delta \kappa} > 0$ for all regimes. The indirect positive effect of civil monitoring on output always dominates its detrimental effect in Regime 1 and $\forall \kappa < \kappa^* = 0.7$ in Regime 2, while the reverse is true in regime 3 for all κ . The speed of convergence decreases substantially with the increase of κ in all three regimes. Furthermore, comparing the two moods of the game, we see that the cooperative regime displays much faster convergence than the competitive regimes.

The second case involves a simple variation of β in two different situations when κ is either small or large. Note that the parameter β does not enter the model of Regime 1 in which there is no government. So we study the local impact of a small variation of β in Regime 2 and Regime 3. If κ is small (Let us take $\kappa = 0.05$) and the remaining parameters stay unchanged, the variations of β are insufficient to modify the optimal civil monitoring and consequently, the optimal output and level of institutional quality index. The steady state values remain unchanged. In the Figure 4, we graphically illustrate the null impact of a simple variation of β on the optimal effort of civil society. The interesting case is when κ is high. To be in line with the first numerical example involving a simple variation of κ , we take $\kappa = 0.45$

Table 5: Effect of variation of β in Regime 2 with high κ

| β | w^* | I^* | λ^* | y^* | v_2 |
|---------|-------|-------|-------------|-------|--------|
| 0.003 | 0.204 | 1.021 | 0.412 | 0.814 | -0.210 |
| 0.3 | 0.199 | 0.993 | 0.415 | 0.797 | -0.216 |
| 0.5 | 0.196 | 0.978 | 0.416 | 0.787 | -0.221 |

It can easily be shown that an increase of β , other factors staying unchanged, will decrease the optimal civil monitoring (even if the impact is small as reported in Figure 5) and the optimal level of institutional quality index. The change in institutional quality indirectly lowers the output through the TFP effects, while the change in the optimal civil monitoring means that more labor is devoted to the production sector (which in turn enhances the output). The steady state value of the output decreases as β increases, thus the TFP effects always dominate.

Table 6: Effects of variation of β in Regime 3 with high κ

| β | w^* | I^* | λ^* | y^* | v_3 |
|---------|-------|-------|-------------|-------|--------|
| 0.003 | 0.094 | 0.472 | 0.323 | 0.428 | -5.304 |
| 0.3 | 0.099 | 0.497 | 0.266 | 0.448 | -3.987 |
| 0.5 | 0.103 | 0.517 | 0.241 | 0.465 | -3.353 |

In this case the civil monitoring, the institutional quality and the output all increase with the positive variation of β coupled with the high scale factor of corruption κ . Again, the TFP effects dominates. Figure 6 exhibits the variation of effort w associated with a variation of β . Comparing the speed of convergence to the long run equilibrium, we realize that it is always greater in a cooperative regime than in a competitive one.

5 Conclusion

In this paper, we analyzed the dynamic interaction between civil society and government in a representative developing country. We distinguished three regimes: the interaction between an active or loyal civil society and passive government; an active civil society and an oppressive government; and the cooperative arrangement. We demonstrated analytically that all three regimes display a unique stable equilibrium (interior solution). In the first regime, the corner solution of zero effort may occur if civil society chooses to be loyal. The numerical experiments indicate that in three regimes, the higher the scale factor of corruption in the economy, the bigger the civil monitoring. Civil monitoring is low and the speed of convergence to the long run equilibrium is always faster in the cooperative regime. Furthermore, the total factor productivity effects always dominate the direct detrimental effect of civil society on production in Regime 1 and for $\kappa < 0.7$ in Regime 2. The reverse case appears under cooperation. The change in the aversion parameter of rental variations is neutral if the scale of corruption is too low in the that sense it does not modify the civil monitoring and consequently output. However, an increase in the aversion parameter of variations of rent coupled with high governmental capacity of creating rent will decrease civil

monitoring in Regime 2. Conversely, it will increase civil monitoring in Regime 3. TFP effects will always dominate.

Our results are based on open-loop strategies. A further issue will be to characterize the feedback outcome strategies of this differential game, ensuring that optimal solution is a subgame perfect equilibrium which guarantees the time consistency property despite the possible loss of analytical tractability.

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Appendix A

A representative consumer

$$\begin{aligned} & \text{Max}_{w_t} \int_0^\infty \exp(-\rho t) U(C_t) dt, \text{ subject to Equation (1)} \\ & \text{with } 0 \leq w_t \leq 1 \end{aligned}$$

Let us denote the utility function $U(C(w, I))$ by $M(w, I)$ and the state equation by $\dot{I} = f(w, I)$ since the control variable is w and the state variable is I . Then, the current value hamiltonian and the augmented lagrangian function can be respectively written as

$$\begin{aligned} H &= M(w, I) + \lambda f(w, I) \\ & \text{and} \\ L &= M(w, I) + \lambda f(w, I) + \mu_{1t} w_t + \mu_{2t} (1 - w_t) \end{aligned}$$

By definition, necessary conditions of the Maximum Principle (5)-(9) are also sufficient under the **Mangasarian sufficiency theorem** iff both M and f are Joint Concave in (w, I) and $\lambda(t) \geq 0, \forall t$. In other words, H is concave in (w, I) i.e the Hessian Matrix associated to H is definite negative.

Following Chiang Alpha (1992), we don't need to check the condition on λ since the function f is linear in w and in I . Furthermore, the constraint qualification is satisfied since the inequality constraint, g_i for $i = (1; 2)$, are all linear in w . For our case, we have to verify only the concavity condition of the function M with respect to w and I . Preliminary, we have:

$$\begin{aligned} M_w &= -U_c A(\phi_w F + (1 - \phi(w)) F_L) \\ M_I &= U_c (1 - \phi(w)) F_{A_I} \end{aligned}$$

Let us now compute the hessian matrix associated to $M(w, I)$ as follows:

$$\begin{pmatrix} M_{ww} & M_{wI} \\ M_{Iw} & M_{II} \end{pmatrix}$$

Where

$$\begin{aligned} M_{ww} &= U_{cc}[A(\phi_w F + (1 - \phi(w))F_L)]^2 - U_c A[\phi_{ww} F - 2\phi_w F_L - (1 - \phi(w))F_{LL}] \\ M_{wI} &= M_{Iw} = -(U_{cc}(1 - \phi(w))A_I F - U_c A_I)(\phi_w F + (1 - \phi(w))F_L) \\ M_{II} &= -U_{cc}[(1 - \phi(w))A_I F]^2 + U_c(1 - \phi(w))F A_{II} \end{aligned}$$

According to a standard utility function $U(C)$ and a standard production function, we can conclude that $M_{ww} < 0$ iff $\phi_{ww} \geq 0$ and $M_{II} < 0$ iff $A_{II} \leq 0$. Furthermore, we see that $M_{ww}M_{II} - (M_{wI})^2 > 0$, meaning that the discriminant is positive. Thus, the hessian matrix above is definite negative and the FOCs (5) - (9) give a maximum.

Appendix B

Government

$$Max_{x_t} \int_0^\infty exp(-\rho t)[U(G_t) - g(x_t)]dt \text{ subject to Equation (1)}$$

where $G_t = \phi(x_t, w_t)Y_t$.

The current hamiltonian associated to the control problem of the Government is

$$H = U(\phi(x_t, w_t)Y_t) - g(x_t) + \eta(bt - \delta I_t) \text{ where } \eta \text{ is the costate variable}$$

The necessary conditions (22) - (24) are sufficient for maximum principle iff H is concave in (x, I) meaning that the associated hessian matrix is definite negative. Computing the hessian matrix associated to H , we get:

$$\begin{pmatrix} H_{xx} & H_{xI} \\ H_{Ix} & H_{II} \end{pmatrix}$$

Where

$$\begin{aligned} H_{xx} &= U_{GG}(\phi_{xy})^2 + U_G \phi_{xxy} - g_{xx} \\ H_{II} &= U_{GG}(\phi A_I F)^2 + U_G \phi A_{II} F \\ H_{Ix} &= H_{xI} = (U_{GG} \phi A_I F + U_G) \phi_x A_I F \end{aligned}$$

With the convex cost function $g(x)$ and a standard utility function $U(G)$, it can easily be shown that $H_{xx} < 0$ iff $\phi_{xx} \leq 0$ and $H_{II} < 0$ iff $A_{II} \leq 0$. Furthermore, the relation $H_{xx}H_{II} - (H_{Ix})^2 > 0$ is verified. Then, the hessian matrix associated to H is definite negative meaning that the FOCs (23) - (25) give a Maximum.

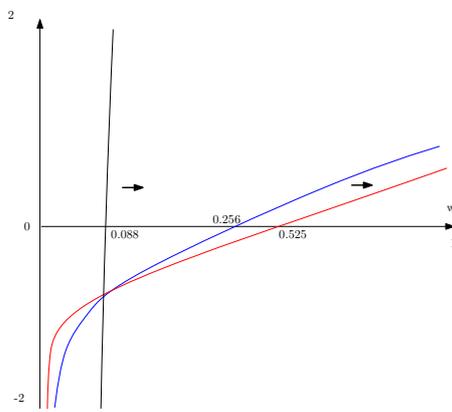
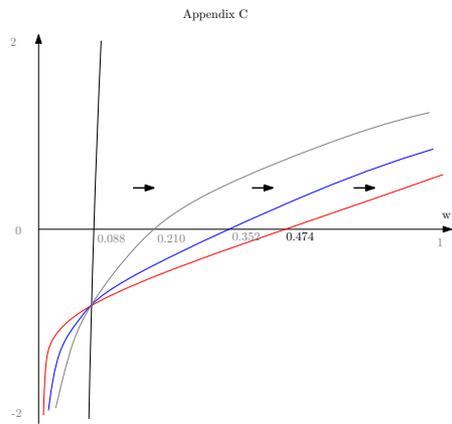


Figure 1: Variation of κ in Regime 1 and 2

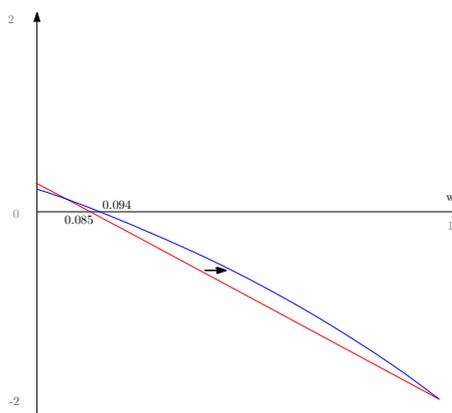


Figure 2: Variation of κ in Regime 3

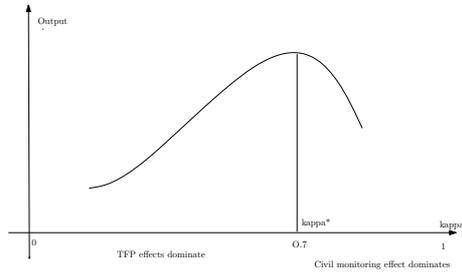


Figure 3: At κ^* , TFP effects and civil monitoring effects totally compensate in Regime 2

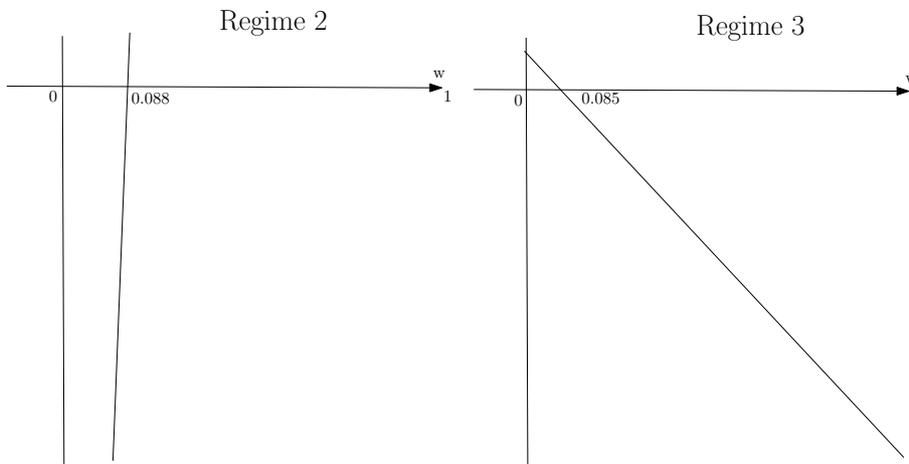


Figure 4: Variation of β when κ is small

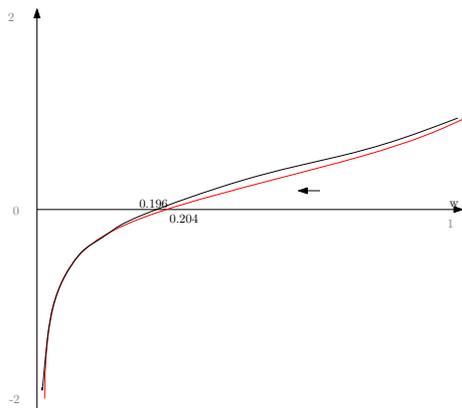


Figure 5: Variation of β when κ is high in Regime 2

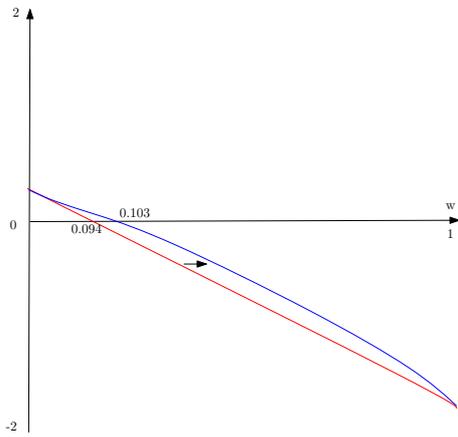


Figure 6: Variation of β when κ is high in Regime 3